

$$\langle x^2, 1 \rangle = \int_0^1 x^2 dx = \frac{1}{3}$$

$$\|1\|^2 = 1$$

$$\langle x^2, v_2 \rangle = \langle x^2, x - \frac{1}{2} \rangle = \int_0^1 x^3 - \frac{x^2}{2} dx = \frac{1}{12}$$

$$\|v_2\|^2 = \int_0^1 (x - \frac{1}{2})^2 dx = \frac{1}{12}$$

$$\Rightarrow v_3 = x^2 - \frac{\langle x^2, 1 \rangle}{\|1\|^2} \cdot 1 - \frac{\langle x^2, v_2 \rangle}{\|v_2\|^2} \cdot v_2 =$$

$$= x^2 - \frac{1}{3} - 1 \cdot (x - \frac{1}{2}) =$$

$$= \underline{\underline{x^2 - x + \frac{1}{6}}}$$

$$v_3' = \frac{v_3}{\|v_3\|} = \frac{1}{6\sqrt{5}} (x^2 - x + \frac{1}{6}) //$$

$$\|v_3\| = \left(\int_0^1 (x^2 - x + \frac{1}{6})^2 dx \right)^{1/2} = \sqrt{\frac{1}{180}} = \frac{1}{6\sqrt{5}}$$