

גבול של פונקציה – הגדרת הגבול לפי קושי

$$\Leftrightarrow \lim_{x \rightarrow x_0} f(x) = L \bullet$$

$$\forall \varepsilon > 0 \exists \delta > 0 : 0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

$$\Leftrightarrow \lim_{x \rightarrow \infty} f(x) = L \bullet$$

$$\forall \varepsilon > 0 \exists M > 0 : x > M \Rightarrow |f(x) - L| < \varepsilon$$

$$\Leftrightarrow \lim_{x \rightarrow -\infty} f(x) = L \bullet$$

$$\forall \varepsilon > 0 \exists M < 0 : x < M \Rightarrow |f(x) - L| < \varepsilon$$

$$\Leftrightarrow \lim_{x \rightarrow x_0} f(x) = \infty \bullet$$

$$\forall M > 0 \exists \delta > 0 : 0 < |x - x_0| < \delta \Rightarrow f(x) > M$$

$$\Leftrightarrow \lim_{x \rightarrow x_0} f(x) = -\infty \bullet$$

$$\forall M < 0 \exists \delta > 0 : 0 < |x - x_0| < \delta \Rightarrow f(x) < M$$

$$\Leftrightarrow \lim_{x \rightarrow \infty} f(x) = \infty \bullet$$

$$\forall M > 0 \exists \delta > 0 : x > \delta \Rightarrow f(x) > M$$

$$\Leftrightarrow \lim_{x \rightarrow \infty} f(x) = -\infty \bullet$$

$$\forall M < 0 \exists \delta > 0 : x > \delta \Rightarrow f(x) < M$$

$$\Leftrightarrow \lim_{x \rightarrow -\infty} f(x) = \infty \bullet$$

$$\forall M > 0 \exists \delta < 0 : x < \delta \Rightarrow f(x) > M$$

$$\Leftrightarrow \lim_{x \rightarrow -\infty} f(x) = -\infty \bullet$$

$$\forall M < 0 \exists \delta < 0 : x < \delta \Rightarrow f(x) < M$$