

$$1 = x \frac{1}{x} \leq x \left[\frac{1}{x} \right] \leq x \left(\frac{1}{x} + 1 \right) = 1 + x$$