פתרון תרגיל 3 אינפי2 מדמח

תזכורת:

$$\sin^2 x = \frac{1-\cos 2x}{2} \quad , \quad \cos^2 x = \frac{1+\cos 2x}{2} \quad , \quad \sin x \cos x = \frac{1}{2}\sin 2x$$

סעיף 1:

$$\int \sin^4 x \cos^2 x \,\mathrm{d}x = \int \sin^2 x \cdot (\sin x \cos x)^2 \,\mathrm{d}x =$$

$$= \int \frac{1-\cos 2x}{2} \cdot \left(\frac{\sin 2x}{2}\right)^2 \mathrm{d}x = \frac{1}{8}\int \sin^2 2x \,\mathrm{d}x - \frac{1}{8}\int \sin^2 2x \cos 2x \,\mathrm{d}x$$

לאינטגרל ראשון:

$$\int \sin^2 2x \,\mathrm{d}x = \frac{1}{2}\int (1-\cos 4x)\,\mathrm{d}x = \frac{x}{2} - \frac{1}{8}\sin 4x + C$$

לאינטגרל שני:

$$\int \sin^2 2x \cos 2x \,\mathrm{d}x = \left[\begin{array}{c} \sin 2x = t \\ 2\cos 2x \,\mathrm{d}x = \mathrm{d}t \end{array}\right] = \frac{1}{2}\int t^2 \,\mathrm{d}t$$

$$= \frac{1}{6}t^3 + C = \frac{1}{6}\sin^3 2x + C$$

והתשובה הסופית היא

$$\int \sin^4 x \cos^2 x \,\mathrm{d}x = \frac{x}{16} - \frac{1}{64}\sin 4x - \frac{1}{48}\sin^3 2x + C$$

סעיף 2:

$$\int \cos^5 x \,\mathrm{d}x = \int (1-\sin^2 x)^2 \cos x \,\mathrm{d}x = \left[\begin{array}{c} \sin x = t \\ \cos x \,\mathrm{d}x = \mathrm{d}t \end{array}\right] = \dots$$

סעיף 3:

$$\int \cos^4 x \,\mathrm{d}x = \int \left(\frac{1+\cos 2x}{2}\right)^2 \mathrm{d}x = \dots$$

סעיף 4:

$$\int \frac{\cos^3 x}{\sin^2 x}\,\mathrm{d}x = \int \frac{\cos^2 x \cdot \cos x}{\sin^2 x}\,\mathrm{d}x = \left[\begin{array}{c}\sin x = t \\ \cos x\,\mathrm{d}x = \mathrm{d}t\end{array}\right] = \int \frac{1-t^2}{t^2}\,\mathrm{d}t$$

$$= \int \frac{\mathrm{d}t}{t^2} - \int \mathrm{d}t = -\frac{1}{t} - t + C = -\frac{1}{\sin x} - \sin x + C$$

סעיף 5:

$$\int \frac{\mathrm{d}x}{1+\cos^2 x} = \left[\begin{array}{c}\tan x = t \\ \mathrm{d}x = \dfrac{\mathrm{d}t}{1+t^2}\end{array}\right] = \int \frac{\frac{1}{1+t^2}\,\mathrm{d}t}{1+\frac{1}{1+t^2}} = \int \frac{\mathrm{d}t}{t^2+2}$$

$$= \frac{1}{\sqrt{2}} \arctan \frac{t}{\sqrt{2}} + C = \frac{1}{\sqrt{2}} \arctan \frac{\tan x}{\sqrt{2}} + C$$

סעיף 6:

$$\int \tan^3 x\,\mathrm{d}x = \left[\begin{array}{c}\tan x = t \\ \mathrm{d}x = \dfrac{\mathrm{d}t}{1+t^2}\end{array}\right] = \int \frac{t^3}{1+t^2}\,\mathrm{d}t = \int \left(t - \frac{t}{1+t^2}\right)\mathrm{d}t$$

$$= \frac{t^2}{2} - \frac{1}{2}\ln(1+t^2) + C = \frac{1}{2}\tan^2 x - \frac{1}{2}\ln(1+\tan^2 x) + C$$

$$= \frac{1}{2}\tan^2 x + \ln|\cos x| + C$$

סעיף 7:

$$\int \frac{\mathrm{d}x}{\sin^2 x \cos^4 x} = \left[\begin{array}{c}\tan x = t \\ \mathrm{d}x = \dfrac{\mathrm{d}t}{1+t^2}\end{array}\right] = \int \frac{\frac{\mathrm{d}t}{1+t^2}}{\frac{t^2}{1+t^2}\cdot\left(\frac{1}{1+t^2}\right)^2} = \int \frac{(1+t^2)^2}{t^2}\,\mathrm{d}t$$

$$= \int \left(\frac{1}{t^2} + 2 + t^2\right)\mathrm{d}t = -\frac{1}{t} + 2t + \frac{t^3}{3} + C$$

$$= -\cot x + 2\tan x + \frac{1}{3}\tan^3 x + C$$

סעיף 8:

$$\int \frac{\mathrm{d}x}{3+\cos x} = \left[\ \tan\tfrac{x}{2} = t\ \right] = \int \frac{\frac{2\,\mathrm{d}t}{1+t^2}}{3+\frac{1-t^2}{1+t^2}} = \int \frac{\mathrm{d}t}{t^2+2}$$

$$= \frac{1}{\sqrt{2}} \arctan \frac{t}{\sqrt{2}} + C = \frac{1}{\sqrt{2}} \arctan \frac{\tan\frac{x}{2}}{2} + C$$