

$$\begin{matrix} 5-6 & \delta'271 \\ (\delta'120) & \delta'271 \end{matrix}$$

$$2y'' - 5y' + 2y = 0 \quad (1)$$

$$2k^2 - 5k + 2 = 0 \quad : k''$$

$$k_{1,2} = \frac{5 \pm 3}{2} = \begin{matrix} 4 \\ -1 \end{matrix}$$

⊆ p'ile p'enn p'ente u'k'ip \*

$$y = C_1 \cdot e^{4x} + C_2 \cdot e^{-x}$$

$$y'' - 4y' + 5y = 0 \quad (2)$$

$$k^2 - 4k + 5 = 0 \quad : k''$$

$$k_{1,2} = \frac{4 \pm 2i}{2} = 2 \pm i$$

⊆ p'271 p'ente u'k'ip \*

$$y = e^{2x} \cdot (C_1 \cos x + C_2 \sin x)$$

$$p'' = \dots$$

(3)

$$K_{1,2} = \frac{-6 \pm 0}{18} = -\frac{1}{3}$$

⇒ מקבילי ע"ע נרדף

$$y = e^{-\frac{1}{3}x} (C_1 + C_2 \cdot x)$$

$$y'' - 2y' + 2y = 0 \quad (4)$$

$$\circ \quad K^2 - 2K + 2 = 0 \quad \underline{\text{ג"ו}}$$

$$K_{1,2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

⇒ מקבילי ע"ע נרדף

$$y = e^x (C_1 \cos x + C_2 \sin x)$$

$$\circ \quad 9y'' - 12y' + 4y = 0 \quad (5)$$

$$9K^2 - 12K + 4 = 0 \quad \underline{\text{ג"ו}}$$

$$K_{1,2} = \frac{12 \pm 0}{18} = \frac{2}{3}$$

⇒ מקבילי ע"ע נרדף

$$y = e^{\frac{2}{3}x} (C_1 + C_2 x)$$

$$9y'' - 3y' - 2y = 0 \quad (1)$$

$$y(0) = 3, \quad y'(0) = 1$$

$$9k^2 - 3k - 2 = 0 \quad \text{: } \underline{k''}$$

$$k_{1,2} = \frac{3 \pm 9}{18} = \begin{cases} \frac{2}{3} \\ -\frac{1}{3} \end{cases}$$

$\Leftarrow$   $\rho$  ז"ל  $\rho$  "למ"ל  $\rho$  ז"ל 2  $\rho$  ז"ל \*  $\rho$  ז"ל \*

$$y = C_1 \cdot e^{\frac{2}{3}x} + C_2 \cdot e^{-\frac{1}{3}x} \quad \Leftarrow \underline{\rho} \text{ ז"ל} \rho$$

$\rho$  ז"ל  $\rho$  ז"ל  $\rho$  ז"ל  $\rho$  ז"ל  $\rho$  ז"ל \*

$$\begin{cases} y(0) = 3 \\ y'(0) = 1 \end{cases}$$

$$y' = \frac{2}{3}C_1 e^{\frac{2}{3}x} - \frac{1}{3}C_2 e^{-\frac{1}{3}x}$$

$$\begin{aligned} y(0) = 3 &\Rightarrow \begin{cases} C_1 + C_2 = 3 \\ \frac{2}{3}C_1 - \frac{1}{3}C_2 = 1 \end{cases} \Rightarrow \begin{cases} C_1 = 2 \\ C_2 = 1 \end{cases} \\ y'(0) = 1 &\Rightarrow \end{aligned}$$

$$y = 2 \cdot e^{\frac{2}{3}x} + e^{-\frac{1}{3}x} \quad \Leftarrow \underline{\rho} \text{ ז"ל} \rho$$

$$y'' - 2\sqrt{5}y' + 5y = 0$$

$$y(0) = 0, y'(0) = 3$$

$$k^2 - 2\sqrt{5}k + 5 = 0 \quad : k''$$

$$k_{1,2} = \frac{2\sqrt{5} \pm 0}{2} = \sqrt{5}$$

← נקודת זרימה יחידה  $\sqrt{5}$  \*  
 $y = e^{x\sqrt{5}} (C_1 + C_2 x)$   
יציב

← נקודת זרימה  $\sqrt{5}$  \*  
 $y' = \sqrt{5} \cdot e^{x\sqrt{5}} (C_1 + C_2 x) + C_2 \cdot e^{x\sqrt{5}} =$   
 $= e^{x\sqrt{5}} [\sqrt{5} \cdot C_1 + (x\sqrt{5} + 1)C_2]$

•  $y(0) = 0 \Rightarrow C_1 = 0$   
 $y'(0) = 3 \Rightarrow C_2 = 3$

•  $y = 3x \cdot e^{x\sqrt{5}}$  ← נקודת זרימה  $\sqrt{5}$  \*

$$k^2 - \sqrt{2}k + 1 = 0 \quad : k'' = 0$$

$$k_{1,2} = \frac{\sqrt{2} \pm i\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \pm i \frac{\sqrt{2}}{2}$$

⊆ פ' ופ"ו נפרדים ונבדלים \*  

$$y = e^{\frac{x\sqrt{2}}{2}} \left( C_1 \cos \frac{x\sqrt{2}}{2} + C_2 \sin \frac{x\sqrt{2}}{2} \right)$$

~~נפרדים~~

: (נ) נפרד (3M) \*  

$$y' = \frac{\sqrt{2}}{2} e^{\frac{x\sqrt{2}}{2}} \cdot \left( C_1 \cos \frac{x\sqrt{2}}{2} + C_2 \sin \frac{x\sqrt{2}}{2} \right) +$$

$$+ e^{\frac{x\sqrt{2}}{2}} \cdot \left( -\frac{\sqrt{2}}{2} C_1 \sin \frac{x\sqrt{2}}{2} + \frac{\sqrt{2}}{2} C_2 \cos \frac{x\sqrt{2}}{2} \right) =$$

$$= \frac{\sqrt{2}}{2} e^{\frac{x\sqrt{2}}{2}} \cdot \left[ C_1 \left( \cos \frac{x\sqrt{2}}{2} - \sin \frac{x\sqrt{2}}{2} \right) + C_2 \left( \sin \frac{x\sqrt{2}}{2} + \cos \frac{x\sqrt{2}}{2} \right) \right]$$

$$y(0) = \sqrt{2} \Rightarrow \begin{cases} C_1 = \sqrt{2} \\ \frac{\sqrt{2}}{2} C_1 + \frac{\sqrt{2}}{2} C_2 = 0 \end{cases} \Rightarrow \begin{cases} C_1 = \sqrt{2} \\ C_2 = -\sqrt{2} \end{cases}$$

$$y = \sqrt{2} \cdot e^{\frac{x\sqrt{2}}{2}} \cdot \left( \cos \frac{x\sqrt{2}}{2} - \sin \frac{x\sqrt{2}}{2} \right)$$

↑

(\*)

$$y'' - 6y' + 25y = 2\sin x + 3\cos x \quad (1)$$

\* נרמזי מיליב ל - המוליה  
המאונני -

$$y'' - 6y' + 25y = 0$$

$$k^2 - 6k + 25 = 0 \quad \text{מ"ל}$$

$$k_{1,2} = \frac{6 \pm \sqrt{d}}{2} = 3 \pm 4i$$

\* קיבלנו להל' מרובקי'  $\Leftrightarrow$

$$\tilde{y} = e^{3x} \cdot (C_1 \cos 4x + C_2 \sin 4x)$$

\* מרובן בלי לה המוליה המאונני

מבוא:

אם אלה י'אן לה המוליה ל' המאונני -  
מ' מקנ'א' קב'ע'א' ה'א' מהצורה:

$$f(x) = e^{\alpha x} \cdot [P_n(x) \cos \beta x + Q_n(x) \sin \beta x]$$

$\alpha, \beta$  - קב'ע'א' -  $P_n(x), Q_n(x)$  - מ'נ'ו'א'א' ק'ס'א  
ה' ה' בה'מ'ה.

מ'נ'ו'א'א' ק'ס'א - המוליה ב'לי  
המאונני - הצורה:

$$y = e^{\alpha x} \cdot [P(x) \cos \beta x + Q(x) \sin \beta x]$$

$$f(x) = 2\sin\frac{1}{2}x + 3\cos x$$

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$$2 + 1\beta = i \Leftrightarrow \begin{cases} \alpha = 0 \\ \beta = 1 \end{cases}$$

$$f = 0 \Leftrightarrow f'' = 0 \quad \text{for } x = 0 \text{ and } x = \pi$$

,  $m = n = l = 0$

∴  $f(x) = A \cos x + B \sin\frac{1}{2}x$

$$u = A \cos x + B \sin\frac{1}{2}x$$

$$u' = -A \sin\frac{1}{2}x + B \cos x$$

$$u'' = -A \cos x - B \sin\frac{1}{2}x$$

∴  $f''(x) = -A \cos x - B \sin\frac{1}{2}x + 6A \sin\frac{1}{2}x - 6B \cos x + 25A \cos x + 25B \sin\frac{1}{2}x = 2\sin\frac{1}{2}x + 3\cos x$

$$-A \cos x - B \sin\frac{1}{2}x + 6A \sin\frac{1}{2}x - 6B \cos x + 25A \cos x + 25B \sin\frac{1}{2}x = 2\sin\frac{1}{2}x + 3\cos x$$

$$(6A + 24B) \sin\frac{1}{2}x = 2\sin\frac{1}{2}x$$

$$(24A - 6B) \cos x = 3\cos x$$

$$\begin{cases} 6A + 24B = 2 \\ 24A - 6B = 3 \end{cases} \Rightarrow \begin{cases} 24A + 96B = 8 \\ 24A - 6B = 3 \end{cases} \Rightarrow \begin{cases} B = \frac{1}{10} \\ A = \frac{1}{10} \end{cases}$$

$$u = \frac{14}{10} \cos x + \frac{5}{10} \sin\frac{1}{2}x$$

: לכן (\*) ה'ב'ן פונקציה, ו'ב'ן

$$y = \tilde{y} + u = e^{3x} (C_1 \cos 4x + C_2 \sin 4x) + \frac{14}{102} \cos x + \frac{5}{102} \sin x$$

(\*)  $y'' - 6y' + 8y = 3x^2 + 2x + 1$

○  $y'' - 6y' + 8y = 0$  : ה'ב'ן  
 $k^2 - 6k + 8 = 0$  : ה'ב'ן

$$k_{1,2} = \frac{6 \pm 2}{2} = \begin{matrix} 4 \\ -1 \end{matrix}$$

$$\tilde{y} = C_1 e^{4x} + C_2 e^{-x}$$

← ה'ב'ן פונקציה

○  $f(x) = 3x^2 + 2x + 1$

$$\lambda + i\beta = 0 \Leftrightarrow \begin{cases} \lambda = 0 \\ \beta = 0 \end{cases}$$

$m = l = 2, \Gamma = 0 \Leftrightarrow$  ה'ב'ן ה'ב'ן נ'כ' - 0

: ה'ב'ן (\*) ה'ב'ן פונקציה

$$u = Ax^2 + Bx + C$$

$$u' = 2Ax + B$$



$$2A - 12Ax - 6B + 8Ax^2 + 8Bx + 8C = 3x^2 + 2x$$

$$8Ax^2 + (8B - 12A)x + (2A - 6B + 8C) = 3x^2 + 2x$$

⇓

$$8A = 3 \Rightarrow A = \frac{3}{8} //$$

$$8B - 12A = 8B - \frac{36}{8} = 2 \Rightarrow B = \frac{13}{16} //$$

$$2A - 6B + 8C = \frac{6}{8} - \frac{39}{8} + 8C = 1 \Rightarrow C = \frac{41}{64} //$$

$$u = \frac{3}{8}x^2 + \frac{13}{16}x + \frac{41}{64}$$

$$y = \tilde{y} + u = C_1 e^{4x} + C_2 e^{-x} + \frac{1}{64} (24x^2 + 52x + 41)$$

Prüfung

$$(*) \quad y'' + 2y' = 2 + e^{-x}$$

$$y'' + 2y' = 0$$

$$k^2 + 2k = 0$$

$$k(k+2) = 0$$

⇓

$$k_1 = 0$$

:-'J2/N17

:-k''g

$$f(x) = f_1(x) + f_2(x) = 2 + e^{-x}$$

$$f_1(x) = 2$$

$$L+1)^{\beta} = 0 \Leftrightarrow \begin{cases} \alpha = 0 \\ \beta = 0 \end{cases}$$

$r=1 \Leftrightarrow 1$  יציב ל"ג  $\ell$   $m=h=l=0$

$$\boxed{u_1 = A \cdot x}$$

$$\bullet f_2(x) = e^{-x}$$

$$L+1)^{\beta} = -1 \begin{cases} \alpha = -1 \\ \beta = 0 \end{cases}$$

$m=h=l=0, r=0 \Leftrightarrow$  ל"ג  $\ell$   $m=h=l=0$

$$\boxed{u_2 = B \cdot e^{-x}}$$

$\bullet$  יציב ל"ג  $\ell$   $m=h=l=0$

$$u = u_1 + u_2 = \boxed{Ax + B \cdot e^{-x}}$$

$$u' = A - B \cdot e^{-x}$$

$$u'' = B \cdot e^{-x}$$

$$B e^{-x} + 2A - 2B \cdot e^{-x} = 2 + e^{-x}$$

$$u = x - e^{-x}$$

$$y = \tilde{y} + u = C_1 + C_2 e^{-2x} + x - e^{-x}$$

(x) (a) (b) (c) (d)

$$y'' - 9y' + 20y = x^2 \cdot e^{4x}$$

$$y'' - 9y' + 20y = 0 \quad \text{Charakteristisches Polynom}$$

$$k^2 - 9k + 20 = 0 \quad \text{Quadrat}$$

$$k_{1,2} = \frac{9 \pm 1}{2} = \begin{matrix} 5 \\ -4 \end{matrix}$$

$$\tilde{y} = C_1 \cdot e^{5x} + C_2 \cdot e^{-4x}$$

$$f(x) = x^2 \cdot e^{4x}$$

$$L + \beta = 4 \quad \Leftrightarrow \begin{cases} L = 4 \\ \beta = 0 \end{cases}$$

$$\Gamma = 1 \quad \Leftrightarrow \text{1. Ableitung} \quad \text{2. Ableitung} \quad \text{3. Ableitung} \quad \text{4. Ableitung} \quad \text{5. Ableitung}$$
$$\Leftrightarrow \beta = 2 = 2$$

$$u = x \cdot (Ax^2 + Bx + C) \cdot e^{4x} = (Ax^3 + Bx^2 + Cx) e^{4x}$$

$$u' = (3Ax^2 + 2Bx) \cdot e^{4x} + (4Ax^3 + 4Bx^2 + 4Cx) \cdot e^{4x}$$

$$\begin{aligned}
 u'' &= (12Ax^2 + (6A+8B)x + 2B+4C) \cdot e^{4x} + \\
 &+ (16Ax^3 + (12A+16B)x^2 + (8B+16C)x) e^{4x} \\
 &= [16Ax^3 + (24A+16B)x^2 + (6A+16B+16C)x \\
 &+ (2B+4C)] \cdot e^{4x}
 \end{aligned}$$

: (2) (a) → 23)

$$\begin{aligned}
 & [16Ax^3 + (24A+16B)x^2 + (6A+16B+16C)x \\
 & + (2B+4C)] e^{4x} - 9[4Ax^3 + (3A+4B)x^2 + \\
 & + (2B+4C)x] e^{4x} + 20(Ax^3 + Bx^2 + Cx) e^{4x} \\
 & = x^2 \cdot e^{4x}
 \end{aligned}$$

: (3) (a) → 23)  $e^{4x} \rightarrow$  23) \*

$$\begin{aligned}
 x^3: & 16A - 36A + 20A = 0 \Rightarrow 0 = 0 \\
 = &
 \end{aligned}$$

$$\begin{aligned}
 x^2: & 24A + 16B - 27A - 36B + 20B = 1 \\
 = & -3A = 1 \Rightarrow \boxed{A = -\frac{1}{3}}
 \end{aligned}$$

$$\begin{aligned}
 x: & 6A + 16B + 16C - 18B - 36C + 20C = 0 \\
 = & 6A - 2B = 0
 \end{aligned}$$

$$u = e^{4x} \cdot \left(-\frac{1}{3}x^3 - x^2 + \frac{1}{2}\right) = \boxed{-\frac{e^{4x}}{6} (2x^3 + 6x^2 - 3x + 1)}$$

$$y = \tilde{y} + u = C_1 \cdot e^{5x} + C_2 \cdot e^{4x} - \frac{e^{4x}}{6} (2x^3 + 6x^2 - 3x + 1)$$

$$1) y'' - 4y' + 3y = e^{5x} \quad (*)$$

$$y(0) = 3, y'(0) = 9$$

$$y'' - 4y' + 3y = 0$$

$$k^2 - 4k + 3 = 0$$

$$k_{1,2} = \frac{4 \pm 2}{2} = \begin{matrix} 3 \\ -1 \end{matrix}$$

$$\tilde{y} = C_1 \cdot e^{3x} + C_2 \cdot e^{-x}$$

$$f(x) = e^{5x}$$

$$\alpha + i\beta = 5 \Leftrightarrow \begin{cases} \alpha = 5 \\ \beta = 0 \end{cases}$$

$$m = h = l = 0, r = 0 \Leftrightarrow f(x) = e^{-lx} \text{ für } l = 5$$

$$u = A \cdot e^{5x}$$

$$25Ae^{5x} - 20Ae^{5x} + 3Ae^{5x} = e^{5x} \quad \text{14} \quad \text{: } \frac{1}{p} \quad (x) \rightarrow 23)$$

$$8A \cdot e^{5x} = e^{5x} \Rightarrow \boxed{A = \frac{1}{8}}$$

$$u = \frac{e^{5x}}{8}$$

$$y = \tilde{y} + u = C_1 \cdot e^{3x} + C_2 \cdot e^x + \frac{e^{5x}}{8}$$

(x) A 1/8 p →

$$y' = 3C_1 e^{3x} + C_2 \cdot e^x + \frac{5}{8} e^{5x}$$

$$y(0) = 3 \Rightarrow C_1 + C_2 + \frac{1}{8} = 3 \Rightarrow C_1 + C_2 = \frac{23}{8}$$

$$y'(0) = 9 \Rightarrow 3C_1 + C_2 + \frac{5}{8} = 9 \Rightarrow 3C_1 + C_2 = \frac{67}{8}$$

$$2C_1 = \frac{44}{8} \Rightarrow C_1 = \frac{44}{16} = \frac{11}{4}$$

$$C_2 = \frac{23}{8} - C_1 = \frac{23}{8} - \frac{11}{4} = \frac{1}{8}$$

$$y = \frac{11}{4} e^{3x} + \frac{1}{8} e^x + \frac{e^{5x}}{8}$$

(\*)

$$y'' - 8y' + 16y = e^{4x}$$

$$y(0) = 0, \quad y'(0) = 1$$

הצבה

$$y'' - 8y' + 16y = 0$$

$$k^2 - 8k + 16 = 0 \quad \text{הצבה}$$

$$k_{1,2} = \frac{8 \pm 0}{2} = 4$$

$$\tilde{y} = e^{4x} (c_1 + c_2 x)$$

$$f(x) = e^{4x}$$

$$2 + 1 \cdot 3 = 4 \quad \Leftarrow \begin{cases} \alpha = \\ \beta = \end{cases}$$

$r = 2 \Leftarrow 2$  הצבה  $\alpha = 2, \beta = 0$   $\alpha = 2, \beta = 0$   $\alpha = 2, \beta = 0$

$$u = A \cdot x^2 \cdot e^{4x}$$

$$u' = 2Ax \cdot e^{4x} + 4Ax^2 \cdot e^{4x} = (2Ax + 4Ax^2) \cdot e^{4x}$$

$$u'' = (2A + 8Ax) \cdot e^{4x} + (8Ax + 16Ax^2) \cdot e^{4x} =$$

$$= (16Ax^2 + 16Ax + 2A) \cdot e^{4x}$$

$$(16Ax^2 + 16Ax + 2A)e^{4x} - 8(4Ax^2 + 2Ax)e^{4x} + 16Ax^2 \cdot e^{4x} = e^{4x}$$

$$\underline{x^2}: 16A - 32A + 16A = 0 \Rightarrow 0 = 0$$

$$\underline{x}: 16A - 16A = 0 \Rightarrow 0 = 0$$

$$\underline{1}: 2A = 1 \Rightarrow \boxed{A = \frac{1}{2}}$$

$$u = \underline{\underline{\frac{x^2}{2} \cdot e^{4x}}}$$

$$\boxed{y = \tilde{y} + u = e^{4x}(C_1 + C_2 x) + \frac{x^2}{2} e^{4x}}$$

(\*) in 1/5 11-2

$$y' = 4e^{4x}(C_1 + C_2 x) + C_2 \cdot e^{4x} + x \cdot e^{4x} + 2x^2 \cdot e^{4x}$$

$$y(0) = 0 \Rightarrow \begin{cases} C_1 = 0 \end{cases}$$

$$y'(0) = 1 \Rightarrow \begin{cases} C_2 = 1 \end{cases}$$

$$\boxed{y = e^{4x} \left( \frac{x^2}{2} + x \right)}$$



3) (\*)

$$y'' + y = \cos 3x$$

$$y\left(\frac{\pi}{2}\right) = 4, \quad y'\left(\frac{\pi}{2}\right) = 1$$

$$y'' + y = 0$$

Charakteristisches Polynom

$$k^2 + 1 = 0$$

Charakteristisches Polynom

$$k = \pm i$$

$$\tilde{y} = C_1 \cos x + C_2 \sin x$$

$$f(x) = \cos 3x$$

$$L + LB = 3i \Leftrightarrow \begin{cases} L = \\ B = \end{cases}$$

$$m = l = 0, \quad r = 0 \Leftrightarrow \text{keine Resonanz}$$

$$u = A \cos 3x + B \sin 3x$$

$$u' = -3A \sin 3x + 3B \cos 3x$$

$$u'' = -9A \cos 3x - 9B \sin 3x$$

Einsetzen (\*)

$$-9A \cos 3x - 9B \sin 3x = \cos 3x$$

$$u = -\frac{1}{p} \cos 3x$$

$$y = \tilde{y} + u = C_1 \cos x + C_2 \sin x - \frac{1}{p} \cos 3x$$

(x) le 1/8 p 20

: 1/8 p 20 1/3 p x

$$y' = -C_1 \sin x + C_2 \cos x + \frac{3}{p} \sin 3x$$

$$\begin{aligned} y\left(\frac{\pi}{2}\right) = 4 &\Rightarrow \begin{cases} C_2 = 4 \\ -C_1 - \frac{3}{8} = 1 \end{cases} \Rightarrow \begin{cases} C_1 = -\frac{11}{8} \\ C_2 = 4 \end{cases} \\ y'\left(\frac{\pi}{2}\right) = 1 &\Rightarrow \end{aligned}$$

$$y = -\frac{11}{8} \cos x + 4 \sin x - \frac{1}{8} \cos 3x$$

le 1/8 p 20

4)

(x)

$$2y'' - y' = 1$$

$$y(0) = 0, y'(0) = 1$$

$$2y'' - y' = 0$$

le 1/8 p 20

$$\tilde{y} = C_1 + C_2 \cdot e^{\frac{1}{2}x}$$

$$f(x) = 1$$

$$2 + 1/\beta = 0 \Leftrightarrow \begin{cases} \alpha \\ \beta \end{cases}$$

$\Gamma = 1 \Leftrightarrow 1$  ויבט' N ה"ג ל' ע"ל פ"ג  
m=h=l=0

$$u = Ax$$

$$u' = A$$

$$u'' = 0$$

א' ג' ק -> א' ו' ק' א'

$$-A = 1 \Rightarrow \boxed{A = -1}$$

$$u = -x$$

$$y = \tilde{y} + u = C_1 + C_2 e^{\frac{1}{2}x} - x \Leftrightarrow \begin{cases} \text{א' ב' ג' ד' ה' ו' ז' ח' ט' י' } \\ (x) \text{ ל' } \end{cases}$$

$$y' = \frac{1}{2} C_2 e^{\frac{1}{2}x} - 1$$

$$\begin{cases} y(0) = 0 \Rightarrow C_1 + C_2 = 0 \\ y'(0) = 1 \Rightarrow \frac{1}{2} C_2 - 1 = 1 \end{cases} \Rightarrow \begin{cases} C_1 = -4 \\ C_2 = 4 \end{cases}$$

