9) $$\int \frac{1}{x^2-a^2}\,dx = \begin{cases} -\frac{1}{x}+C & \text{אם}: a=0 \\ \frac{1}{2a}\left(\ln\left|\frac{x-a}{x+a}\right|\right)+C & \text{אחרת}: \circledast \end{cases}$$

$\circledast$ $a\neq 0$: $$\frac{1}{(x-a)(x+a)} = \frac{A}{x-a} + \frac{B}{x+a}$$

$$1 = A(x+a) + B(x-a)$$

$$\Rightarrow \quad A = \frac{1}{2a} \quad ; \quad B = -\frac{1}{2a}$$

$$\Rightarrow \quad \int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\left(\int \frac{dx}{x-a} - \int\frac{dx}{x+a}\right.$$

10) $$\int \frac{x^4}{x^4-1}\,dx = \int 1 + \frac{1}{(x-1)(x+1)(x^2+1)}\,dx$$

$$\frac{1}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Dx+E}{x^2+1}$$

$$A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Dx+E)(x^2-1) = 1$$

נציב $\rightarrow$ $x=-1 \Rightarrow -4B = 1 \Rightarrow B = -\frac{1}{4}$

$x = 1 \Rightarrow 4A = 1 \Rightarrow A = \frac{1}{4}$

השוואת מקדמים
עבור x^3 $\Rightarrow$ $A+B+D=0$

$\frac{1}{4}-\frac{1}{4}+D=0 \Rightarrow D=0$

$x=0 \Rightarrow A-B-E=1 \Rightarrow E=-\frac{1}{2}$

$$\Rightarrow \int\frac{x^4}{x^4+1}\,dx = x + \frac{1}{4}\int\frac{dx}{x-1} - \frac{1}{4}\int\frac{dx}{x+1} - \frac{1}{2}\int\frac{dx}{x^2+1}$$

$$= x + \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\arctan x + C$$

11) $\displaystyle\int \frac{x^3+3x^2+5x+7}{x^2+2}\,dx = \int (x+3)\,dx + \int \frac{3x+1}{x^2+2}\,dx =$

$$\begin{array}{l} \quad x+3 \\ \hline x^3+3x^2+5x+7 \;\big|\; x^2+2 \\ -\;x^3 \qquad\quad +2x \\ \hline \quad -\;3x^2+3x+7 \\ \qquad 3x^2 \qquad +6 \\ \hline \qquad\quad 3x+1 \end{array}$$

$$= \frac{x^2}{2} + 3x + \frac{3}{2}\int \frac{2x}{x^2+2}\,dx + \int \frac{dx}{x^2+2} =$$

$$= \frac{x^2}{2} + 3x + \ln(x^2+2) + \frac{1}{2}\arctan\frac{x}{\sqrt{2}} + C$$

12) $\displaystyle\int \frac{x^5}{(x^3+1)(x^3+8)}\,dx = \begin{bmatrix} t = x^3+1 \\ dt = 3x\,dx \\ t+7 = x^3+8 \\ t-1 = x^3 \end{bmatrix} = \int \frac{(t-1)}{3t(t+7)}\,dt$

$$\frac{t-1}{t(t+7)} = \frac{A}{t} + \frac{B}{t+7}$$

$$t-1 = A(t+7) + Bt$$

[illegible] $\Rightarrow$ $t=0 \Rightarrow -1 = 7A \Rightarrow A = -1/7$

$t=-7 \Rightarrow -8 = -7B \Rightarrow B = 8/7$

$$\Rightarrow \int \frac{x^5}{(x^3+1)(x^3+8)}\,dx = \frac{1}{3}\left(-\frac{1}{7}\right)\int \frac{dt}{t} + \frac{1}{3}\cdot\frac{8}{7}\int \frac{dt}{t+7} =$$

$$= -\frac{1}{21}\ln|t| + \frac{8}{21}\ln|t+7| + C$$

$$= -\frac{1}{21}\ln|x^3+1| + \frac{8}{21}\ln|x^3+8| + C$$

13) $\displaystyle\int \cos 3x \cos 2x\,dx = \int \frac{1}{2}(\cos 5x + \cos x)\,dx =$

זהות:

$\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$

$$= \frac{1}{10}\sin 5x + \frac{1}{2}\sin x + C$$

14) $\int \cos x \cos 2x \cos 4x \, dx = \int \frac{1}{2}(\cos 3x + \cos x) \cos 4x \, dx =$

$= \int \frac{1}{4}(\cos 7x + \cos x + \cos 5x + \cos 3x) \, dx =$

$= \frac{1}{28} \sin 7x + \frac{1}{4} \sin x + \frac{1}{20} \sin 5x + \frac{1}{12} \sin 3x + C$

15) $\int \tan^2 x \, dx = \int \frac{1}{\cos^2 x} - 1 \, dx = \tan x - x + C$