I

חשבון אינפי' 2
תרגיל 2 הפתרון

①

$$\int \frac{dx}{x^2-a^2} = \begin{cases} -\frac{1}{x}+c & a=0 \text{ אם} \\ \frac{1}{2a}\int \frac{1}{x-a}dx - \frac{1}{2a}\int \frac{dx}{x+a}dx \end{cases}$$

$a \neq 0 \quad \frac{1}{(x-a)(x+a)} = \frac{A}{x-a} + \frac{B}{x+a}$

$1 = A(x+a) + B(x-a)$

$A = \frac{1}{2a} \quad B = -\frac{1}{2a}$

$$\Rightarrow \int \frac{dx}{x^2-a^2} = \begin{cases} -\frac{1}{x}+c & a=0 \\ \frac{1}{2a}\ln|x-a| - \frac{1}{2a}\ln|x+a| + c & a \neq 0 \end{cases}$$

②

$$\int \frac{x^3+3x^2+5x+7}{x^2+2}dx$$

$$\begin{array}{l} \quad x+3 \\ x^3+3x^2+5x+7 \; | \; x^2+2 \\ -\underline{x^3+2x} \\ \quad 3x^2+3x+7 \\ -\underline{3x^2+6} \\ \quad 3x+1 \end{array}$$

$$\Rightarrow \int \frac{x^3+3x^2+5x+7}{x^2+2}dx = \int (x+3)dx + \int \frac{3x+1}{x^2+2}$$

$$= \frac{x^2}{2} + 3x + \frac{3}{2}\int \frac{2x}{x^2+2} + \int \frac{dx}{x^2+2}$$

$$= \frac{x^2}{2} + 3x + \frac{3}{2}\ln(x^2+2) + \frac{1}{2}\cdot\sqrt{2}\arctan\frac{x}{\sqrt{2}} + c$$

$$\int \quad dx = \int \quad \frac{}{x^4-1} \quad = x + \int \frac{}{(x-1)(x+1)(x^2+1)}$$

$$\frac{1}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Dx+E}{x^2+1}$$

$$A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Dx+E)(x^2-1) = 1$$

$$x=-1 \Rightarrow \quad -4B = 1 \Rightarrow \boxed{B = -\frac{1}{4}}$$

$$x=1 \Rightarrow \quad 4A = 1 \Rightarrow \boxed{A = \frac{1}{4}}$$

$$x^3: \quad A + B + D = 0$$

$$\frac{1}{4} - \frac{1}{4} + D = 0 \Rightarrow \boxed{D = 0}$$

$$x = 0 \Rightarrow \quad A - B - E = 1$$

$$\frac{1}{4} + \frac{1}{4} - E = 1 \Rightarrow \boxed{E = -\frac{1}{2}}$$

$$\Rightarrow \int \frac{x^4}{x^4-1}\,dx = x + \frac{1}{4}\int\frac{dx}{x-1} - \frac{1}{4}\int\frac{dx}{x+1} - \frac{1}{2}\int\frac{dx}{x^2+1}$$

$$= x + \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| - \frac{1}{2}\arctan x + C$$

4

$$\int \frac{x^5}{(x^3+1)(x^3+8)}\,dx$$

$\boxed{t = x^3+1}$ (נ'ב

$dt = 3x^2\,dx$

$\boxed{\frac{1}{3}dt = x^2\,dx}$

$\boxed{t+7 = x^3+8}$

$\boxed{t-1 = x^3}$

$$\Rightarrow \int \frac{x^5}{(x^3+1)(x^3+8)}\,dx = \int \frac{x^3 \cdot x^2\,dx}{(x^3+1)(x^3+8)}$$

$$= \frac{1}{3}\int \frac{(t-1)\,dt}{t(t+7)}$$

$$\frac{t-1}{t(t+7)} = \frac{A}{t} + \frac{B}{t+7}$$

$$t-1 = A(t+7) + Bt$$

$$t=0 \Rightarrow \quad -1 = 7A \Rightarrow \boxed{A = -\frac{1}{7}}$$

$$t=-7 \Rightarrow \quad -8 = -7B \Rightarrow \boxed{B = \frac{8}{7}}$$

$$\Rightarrow \int \frac{x^5}{(x^3+1)(x^3+8)}\,dx = \frac{1}{3}\cdot\left(-\frac{1}{7}\right)\int \frac{dt}{t}$$

$$+\frac{1}{3}\cdot\frac{8}{7}\int \frac{1}{t+7}\,dt = -\frac{1}{21}\ln|t| + \frac{8}{21}\ln|t+7| + C$$

$$= -\frac{1}{21}\ln|x^3+1| + \frac{8}{21}\ln|x^3+8| + C$$

פונקציה טריגונומטרית

(1)
$$\int \frac{dx}{\cos x + \sin x + 1}$$

הצבה אוניברסלית

$u = \tan\frac{x}{2}$

$\cos x = \frac{1-u^2}{1+u^2}$

$\sin x = \frac{2u}{1+u^2}$

$dx = \frac{2du}{1+u^2}$

$$\Rightarrow \int \frac{2}{(1+u^2)\left[\frac{1-u^2}{1+u^2} + \frac{2u}{1+u^2} + 1\right]}du$$

$$= \int \frac{2}{1-\cancel{u^2}+2u+1+\cancel{u^2}} = \int \frac{2}{2(u+1)}du$$

$$= \int \frac{du}{u+1} = \ln|u+1| + c = \ln\left|\tan\frac{x}{2}+1\right| + c$$

(2)
$$\int \cos 3x \cos 2x \, dx$$

$$= \int \frac{1}{2}\left[\cos 5x + \cos x\right]dx$$

$$= \frac{1}{10}\sin 5x + \frac{1}{2}\sin x + c$$

כלל:
$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$\int \frac{dx}{\sin x} \quad (3)$$

הצבה אוניברסלית

$$\int \frac{2\,du}{(1+u^2)\cdot\frac{2u}{1+u^2}} = \int \frac{du}{u} = \ln|u| + C$$

$$= \ln\left|\tan\frac{x}{2}\right| + C$$

$$\int \cos x \cos 2x \cos 4x \, dx \quad (4)$$

$$\int \frac{1}{2}\left[\cos 3x + \cos x\right]\cos 4x \, dx$$

$$\frac{1}{2}\int \left(\cos 3x \cos 4x + \cos x \cos 4x\right) dx$$

$$\frac{1}{4}\int \left(\cos 7x + \cos x + \cos 5x + \cos 3x\right) dx$$

$$\frac{1}{28}\sin 7x + \frac{1}{4}\sin x + \frac{1}{20}\sin 5x + \frac{1}{12}\sin 3x + C$$

$$\int \tan^2 x \, dx \quad (5)$$

$$= \int \left(\frac{1}{\cos^2 x} - 1\right) dx = \tan x - x + C$$

$$\int \frac{1}{\cos^2 x}\, dx = \tan x + C \quad (6)$$